Failure Modes of Structure Mapping

Diagnosing Misalignment, Overreach, and Projection Artifacts in Cross-Domain Structure

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Abstract

In previous work, we developed a framework in which structure emerges through constraint, stabilizes under operator dynamics, and can be mapped across domains using a common structural schema. However, the presence of structural similarity does not guarantee the validity of such mappings.

In this paper, we identify and analyze common failure modes in structure mapping. These include confusion between representation and structure, projection-induced artifacts, operator mismatch, constraint misalignment, invariant drift, and overextension across domains. We introduce a set of diagnostic tests for evaluating mappings and clarify the conditions under which structural correspondence is meaningful.

This work provides an essential complement to the constructive framework developed in earlier papers, establishing the limits and discipline required for reliable cross-domain reasoning.

1 Introduction: When Connections Mislead

In earlier work, we showed that systems across mathematics, physics, and cognition can be described using a common structural schema involving configuration space, constraint, operator dynamics, invariant structure, and representation.

However, a critical issue arises:

> Not all apparent structural similarities correspond to valid mappings.

The purpose of this paper is to identify where such mappings fail and to provide tools for distinguishing valid structural correspondence from projection artifacts or conceptual error.

2 Valid Structure Mapping

Let two systems be described by:

$$(\Sigma_1, A_1, \Phi_1, I_1, P_1), \quad (\Sigma_2, A_2, \Phi_2, I_2, P_2)$$

A mapping $M : \Sigma_1 \to \Sigma_2$ is considered structurally valid when it preserves:

- admissibility: $M(A_1) \subseteq A_2$,
- operator correspondence: $M(\Phi_1(x)) = \Phi_2(M(x))$ (or an explicitly defined correspondence),
- invariant structure: $M(I_1) \subseteq I_2$,
- observational coherence (when applicable).

Failure to preserve these conditions results in invalid mappings.

3 Failure Mode I: Representation Confusion

Description: Treating different representations as different structures.

- Example: $1/3$ vs $0.333\ldots$
- Error: identifying representation with structure

Diagnostic: Does the mapping depend on notation or encoding?
Resolution: Factor out representation and work with invariant structure.

4 Failure Mode II: Projection Illusion

Description: Inferring structure from observable patterns that arise due to projection.

- Example: apparent randomness from truncated sequences
- Example: loss of cyclic behavior under limited observation

Diagnostic: Does increased precision or access change the observed structure?
Resolution: Model the projection explicitly and analyze its information loss.

5 Failure Mode III: Operator Mismatch

Description: Mapping systems with incompatible transformation rules.

- Example: equating periodic iteration with non-periodic dynamics

Diagnostic:

$$M(\Phi_1(x)) \stackrel{?}{=} \Phi_2(M(x))$$

Resolution: Align operators or restrict the domain of the mapping.

6 Failure Mode IV: Constraint Misalignment

Description: Ignoring differences in admissibility conditions.

- Example: mapping unconstrained systems to constrained systems without adjustment

Diagnostic: Does the mapping preserve admissible sets?
Resolution: Explicitly incorporate or transform constraint structures.

7 Failure Mode V: Invariant Drift

Description: Assuming corresponding invariants where none exist.

- Example: conflating cycles, frequencies, and stability without verification

For example, periodic cycles in modular arithmetic and oscillatory modes in physical systems may appear analogous, but differ in their underlying operator structure.
Diagnostic: Are invariants preserved under mapping?
Resolution: Identify and compare invariant classes explicitly.

8 Failure Mode VI: Domain Overreach

Description: Extending mappings beyond their valid regime.

- Example: finite systems mapped to infinite systems without justification

Diagnostic: Are underlying assumptions consistent across domains?
Resolution: Restrict scope and define domain of validity.

9 Failure Mode VII: Ontological Collapse

Description: Treating analogous structures as identical objects.

- Example: identifying physical fields with their mathematical representations

Diagnostic: Are distinct layers being conflated without a mapping?
Resolution: Maintain separation between layers and define relationships explicitly.

10 Failure Mode VIII: Spurious Generalization

Description: Extrapolating general structure from limited examples.

- Example: assuming all repeating processes share identical structure

Diagnostic: Do counterexamples exist?
Resolution: Test across diverse systems and verify invariance preservation.

11 Diagnostic Framework

To evaluate a proposed mapping, apply the following tests:

1. Representation Test: invariant under change of encoding?
2. Operator Test: does the mapping preserve dynamics?
3. Constraint Test: are admissible sets aligned?
4. Invariant Test: are invariants preserved?
5. Access Test: could the similarity arise from projection?
6. Domain Test: is the mapping applied within the domain where its assumptions hold?

12 Interaction with Limits of Access

Some failures arise not from incorrect reasoning, but from fundamental limits of access.

> Not all structure can be recovered from available representations.

These limitations were explored in earlier work and must be considered when evaluating mappings.

13 Relation to PFI and Closure to Inertia

The Principle of Finite Invariance establishes that meaning depends on invariance under constraint.

From Closure to Inertia describes how structure stabilizes under repeated transformation.

The present work identifies where these conditions fail to hold, and how such failures manifest in practice.

14 Toward Robust Structure Mapping

A structurally valid mapping must:

- preserve constraint and admissibility,
- maintain operator correspondence,
- respect invariant structure,
- acknowledge limits of access,
- specify its domain of validity.

15 Conclusion: Discipline of Mapping

We have identified common failure modes in structure mapping and provided tools for diagnosing them.

Final Statement:

> Structural mapping is meaningful only when constraint, transformation, and invariant structure are preserved; otherwise, apparent similarity reflects projection, not structure.